**ISTA 421 / INFO 521 – Final Project OPTION A: Metropolis-Hastings MCMC Inference of 3D Line**

**Tasks**

1. **Implement Metropolis-Hastings Sampler [10 points]**:
   * Write a Python script to sample from the posterior of the line's endpoints, considering inputs like relative offset ratios, observed points in the camera image plane, noise covariance, and priors over the 3D line segment endpoints.
2. **Draw Samples from Posterior Distribution [6 points]**:
   * Use the script to draw samples from the posterior distribution of the 3D line segment endpoints using data from Camera 1.
   * Generate plots to view sampling progress and describe observed trends.
3. **Find Maximum a Posteriori (MAP) Estimate [4 points]**:
   * Use the generated samples to find the MAP estimate of the line endpoints.
   * Plot the MAP estimate of the line as projected onto the Camera 1 image plane.
4. **Second Camera Perspective [4 points]**:
   * Introduce a second camera and create a plot showing the MAP hypothesis of the endpoints using this new perspective.
   * Compare this plot with the plot for Camera 1.
5. **Metropolis-Hastings Sampling with Both Cameras [10 points]**:
   * Perform sampling using data from both cameras.
   * Find and report the MAP estimate using this combined data.
   * Create plots for each camera projection and evaluate how well they fit the noisy data.

**Submission Requirements**

Submit Python scripts, plots, and a detailed description of trends and results observed.

**Task 1: Implement Metropolis-Hastings Sampler**

Concept:

* **Metropolis-Hastings Algorithm**: It's a Markov Chain Monte Carlo (MCMC) method used to obtain a sequence of random samples from a probability distribution from which direct sampling is difficult. The algorithm generates samples by proposing a move to a new point and accepting or rejecting the move based on a probability criterion.

Implementation Steps:

1. **Initialize Parameters**: Start with initial guesses for the line's endpoints in 3D space.
2. **Iterative Sampling**:
   * Propose a new state (new endpoints) by making a small random change to the current state.
   * Calculate the acceptance ratio based on the likelihood of the new state (how well it explains the observed data) and the prior.
   * Accept or reject the new state based on this ratio.

**Task 2: Draw Samples from Posterior Distribution**

Concept:

* **Posterior Distribution**: It represents the probability of the model parameters given the observed data. In this case, it's the distribution of the line's endpoints after considering the observed 2D points.

Implementation Steps:

1. **Run the Sampler**: Use the Metropolis-Hastings sampler to generate a large number of samples.
2. **Analyze the Samples**: Plot the progression of the samples to understand the convergence and distribution.

**Task 3: Find Maximum a Posteriori (MAP) Estimate**

Concept:

* **MAP Estimate**: This is the set of parameters that maximizes the posterior distribution. It's a point estimate of the model parameters.

Implementation Steps:

1. **Analyze the Generated Samples**: From the samples obtained in Task 2, find the sample that has the highest posterior probability.
2. **Plot the MAP Estimate**: Project this MAP estimate onto the 2D image plane of Camera 1 and plot it.

**Task 4: Second Camera Perspective**

Concept:

* Introducing a second camera involves considering another projection of the 3D line onto a different 2D plane, providing additional data for more accurate inference.

Implementation Steps:

1. **Project MAP Estimate**: Use the camera matrix of the second camera to project the MAP estimate onto its 2D image plane.
2. **Comparison**: Compare the projections from both cameras for consistency and accuracy.

**Task 5: Metropolis-Hastings Sampling with Both Cameras**

Concept:

* **Combined Data Analysis**: This involves using data from both cameras to improve the accuracy of the parameter estimation.

Implementation Steps:

1. **Run the Sampler with Combined Data**: Perform Metropolis-Hastings sampling considering data from both cameras.
2. **Find and Report MAP Estimate**: Analyze the new set of samples to find the MAP estimate.
3. **Create Plots**: Plot the projections for each camera and assess the fit to the noisy observed data.